Direct and indirect empirical statuses (DES and IES) of theoretical symmetries in physics

A Short Introduction *by Valeriya Chasova*

Basic terminology

Symmetry is usually defined as invariance under a transformation.
Theoretical symmetries here mean symmetries of physics.
Ontology of theoretical symmetries is (the study of) their connection with the world.
Empirical statuses of theoretical symmetries are matchings of theoretical symmetries with worldly phenomena. These statuses usually serve to clarify the ontology of theoretical symmetries.

In a wide sense an empirical status is:
- *direct (DES)* if the matching does not pass through intermediary theoretical elements;
- *indirect (IES)* if the matching does pass through intermediary theoretical elements**.**

In a narrow sense:
- *DES* matches theoretical symmetries with empirical symmetries in the world;
- *IES* matches theoretical symmetries with theoretical conservation laws and these are then matched with conservation phenomena in the world.

DES in the narrow sense

Empirical symmetries are symmetries in the world exemplified by four ***recognised examples*** usually defined as follows:

- *Galileo's ship empirical symmetry* is the observable invariance of mechanical phenomena confined to a ship's cabin under its observable boost with respect to the shore;
- *Faraday's cage empirical symmetry* is the observable invariance of electromagnetic phenomena confined to a cage under its observable charging with respect to the ground (manifested by the appearance of sparks at the cage's outer boundaries);
- *Einstein's elevator empirical symmetry* is the observable invariance of weightless (or weighty, if acceleration is added) phenomena confined to an elevator under the addition or removal of a massive body;
- *'t Hooft's beams-splitter empirical symmetry* is the observable change (invariance) of interference pattern in a double-slit experiment under the addition of one (two) half-wave plate(s) (also called phase-shifters) behind the slit(s).

The main controversy in the literature on DES has been over whether only global or also local theoretical symmetries have DES.
Theoretical symmetries are ***global/local*** if they are specified by parameters/functions, or quite equivalently if they are uniform/non-uniform across their domain of application.
Examples of global symmetries: boosts, spatiotemporal translations, spatial rotations, electrostatic potential shifts, phase shifts.
Examples of local symmetries: diffeomorphisms, electromagnetic potential transformations (alone or together with phase transformations).

IES in the narrow sense

Action (S) is an integral of Lagrangian over time or of Lagrangian density over spacetime.
Action is a ***functional***: a function of a function.
Actions and Lagrangians or Lagrangian densities are functions of dependent and independent variables.

Dependent variables are those which depend on independent variables.
Examples of independent variables: $\mathbf{x}$, t, x_μ.
Examples of dependent variables: $\mathbf{A}$, φ, A_μ, ψ.

Functional derivative of an action (δ) encodes variations of (elements of) the action.
An action can be varied with respect to dependent and/or independent variables.
An action can be varied on the bulk and/or at the boundary of its domain of integration.

Group: a set of elements equipped with an operation such that there exists an identity element, every element has an inverse, the operation is associative.
Symmetry group: a group where elements are transformations and which generates some invariance.
Symmetry group of action: the vanishing of the functional derivative of an action (so, invariance of the action) under its variations constituting a group.

↔: bi-implication.

Hamilton's principle (a version of the principle of least action): symmetry groups of actions with respect to the variation of dependent variables in the bulk ↔ the satisfaction of the Euler-Lagrange equations.

The Euler-Lagrange equations (ELEq) are differential equations of a certain form featuring Lagrangians or Lagrangian densities and usually encoding dynamics.
Examples of the ELEq: Maxwell's equations, Einstein's equations, Schrödinger equation in Lagrangian form.

Noether's theorems: symmetry groups of actions (under the variations of dependent and independent variables in the bulk and at the boundary) ↔ some mathematical identities:
- ***Noether's 1st theorem:*** global symmetry groups of actions ↔ 'divergences';
- ***Noether's 2nd theorem:*** local symmetry groups of actions ↔ 'dependencies'.

Conservation laws state that some quantities are preserved over time, i.e. that temporal derivatives of these quantities vanish.
When the ELEq are imposed, 'divergences' and 'dependencies' transform into ***differential conservation laws***.
When boundary conditions are imposed, differential conservation laws transform into ***integral conservation laws***.